# Political Science 597D: Mathematics for Political Science

Thursday 2:30p-3:45p Pond 236

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## 1 Goals of This Course

This class provides a refresher, foundation and overview in mathematical methods useful in coursework and research in political science. One aspiration is to prevent courses from derailing from the flow and substance of the topic being studied every time a new mathematical technique is employed. Another is to create a common expectation of known topics so that they need not be repeated in all classes which use them. Partly this class also relieves a little burden from PS502 and we will try to coordinate some topics so they arrive in time for their application in that class.

A larger goal is to foster a familiarity with the techniques you are most likely to see in articles and research in political science, and allow you to negotiate the technical aspects of the literature without resorting to blind faith. Furthermore it hard to know when you will need a particular tool until the problem presents itself. Even simple exposure to the strategies and tools that exist will help direct you in your future work to more productive attacks in your research and away from roadblocks.

## 2 Problem Sets

There will be problem sets most weeks. Problem sets should be worked on in small groups. Every individual should turn in a copy of solutions. Even if problems are worked on collaboratively, writing the solution by hand, individually, will cement your understanding. Try to turn in homework Friday (to Honaker Mailbox), but attempt the problems ahead of time so questions can be answered in class. There may be a couple short take home quizzes. These are primarily diagnosticboth for you the student and just as much for the instructor. These should be taken individually, without discussion, but with open notes and books. Think of them as short, independently solved, homeworks.

## **3** Acknowledgements

A course of this nature has been developed as a rolling project at Harvard Government department, and many weeks of this class draw heavily from the collected materials of this endeavour. Contributors include Curt Signorino, Ken Scheve, Eric Dickson, Orit Kedar, James Fowler, Kosuke Imai, Jacob Kline, Dan Epstein, Ben Ansell, Ryan Moore, Mike Kellerman, Ellie Powell, Jen Katkin and Patrick Lam.

## 4 What should I do today

You should check the course webpage on Angel has your correct mailing address. You should email me your name, fields of interest, and a description of your previous course-work in maths.

# 5 Chronology of Topics

#### Week 1: Functions I (Spaces and Mappings)

 $\mathbf{R}^1$  and  $\mathbf{R}^n$ , Interval Notation for  $\mathbf{R}^1$ , Introduction to Functions, Domain and Range/Image, Some General Types of Functions, Log, Ln, and e, Solving for Variables.

#### Week 2: Functions II (Distances and Sets)

Summation and Product Notation, Power Law Functions, Distances: Euclidean and Minkowski, Neighborhoods: Intervals, Disks, and Balls, Open/Closed/Compact Sets, Graphing Functions, Solving for Variables.

#### Week 3: Calculus I (Limits)

Finding Roots and Factoring, Piecewise Functions, Limit of a Function.

#### Week 4: Calculus II (Differentiation)

Continuity, Derivatives, Differentiability.

#### Week 5: Calculus III (The Limits of Differentiation)

Composite Functions, Chain Rule, Derivatives of Exponentials, Derivatives of Logarithms, L'Hopital's Rule.

#### Week 6: Calculus IV (Integration)

Indefinite Integrals, Antiderivatives, Riemann Sums, Definite Integrals, Fundamental Theorem of Calculus.

#### Week 7: Calculus V (Challenging Integrals)

Integration by Substitution, Integration by Parts, Numerical Derivatives, Numerical Integrals, Rectangle Rule, Trapezoidal Rule, Simpson's Rule.

#### Week 8: Linear algebra 1 (Vectors and Matrices)

Working with Vectors, Linear Independence, Matrix Algebra, Matrix Multiplication, Vector Products, Square Matrices, Systems of Linear Equations, Method of Substitution, Gaussian Elimination, Gauss-Jordan Elimination.

#### Week 9: Linear Algebra II (Inverses and Determinants)

Matrix Methods for Linear Systems, Rank, Existence of Solutions, Inverse of a Matrix, Linear Systems and Inverses, Determinants, The Determinant Formula for an Inverse, Cramer's Rule.

Week 10: Linear Algebra III (Determinants, Transforms and Eigengoodness) The Determinant Formula for an Inverse, Cramer's Rule, Graphical Representation of Determinants, Matrices as Transformations, Eigenvectors and Eigenvalues.

Week 11: Multivariate Calculus I (Differentiation of Multiple Variables) Partial Derivatives, Gradients, Jacobians, Hessians, Laplacians.

Week 12: Multivariable Optimization I (Optimization of Multiple Variables) Quadratic Forms, Definiteness, Semidefiniteness and Indefiniteness, Maxima, Minima and Saddle Points.

Week 13: Multivariable Optimization II (Numerical Optimization)

The Taylor Series Expansion, The Newton-Raphson Algorithm.

#### Week 14: Proof Strategies I (Formal Logic and Proofs)

Claims, Axioms and Givens, Theorems and the Lesser Relatives, Connectives, Direct Proofs, Proofs by Induction, The Sorites Paradox, Proof by Contradiction, Disproof by Counterexample, Proof of Existence, Proof by Exhaustion, Proof of Uniqueness, Curry-Howard Isomorphism.

The last (15th) week of term will give us a buffer to run overtime on any of the previous topics, otherwise, if we are on pace, we will spend an expanded two weeks on proofs.

# 6 Texts

The bookstore has the books for this course reversed. Gill is the text that we will work from. Simon and Blume is a supplemental text, available in the bookstore. Wackerly is another useful introductory mathematics text. Simon and Blume is strongest in foundational methods for formal theory and economic models. Wackerly is stronger in those methods useful in statistical work.

- Gill, Jeff. 2006. Essential Mathematics for Political and Social Research, Cambridge University Press.
- Simon, Carl P. and Lawrence Blume. 1994. Mathematics for Economists, Norton.
- Wackerly, Dennis, William Mendenhall and Richard Scheaffer. 1996. Mathematical Statistics with Applications, 5th ed. Duxbury Press.

# Academic Dishonesty

The Department of Political Science, along with the College of the Liberal Arts and the University, takes violations of academic dishonesty seriously. Observing basic honesty in one's work, words, ideas, and actions is a principle to which all members of the community are required to subscribe.

All course work by students is to be done on an individual basis unless an instructor clearly states that an alternative is acceptable. Any reference materials used in the preparation of any assignment must be explicitly cited. Students uncertain about proper citation are responsible for checking with their instructor.

In an examination setting, unless the instructor gives explicit prior instructions to the contrary, whether the examination is in class or take home, violations of academic integrity shall consist but are not limited to any attempt to receive assistance from written or printed aids, or from any person or papers or electronic devices, or of any attempt to give assistance, whether the one so doing has completed his or her own work or not.

Lying to the instructor or purposely misleading any Penn State administrator shall also constitute a violation of academic integrity.

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Instructors should be notified as early in the semester as possible regarding the need for reasonable accommodations.